Appendix A

THE THEORY OF FORMATION OF THE K-CORONA

A.1 The general theory on scattering of radiation by free electrons

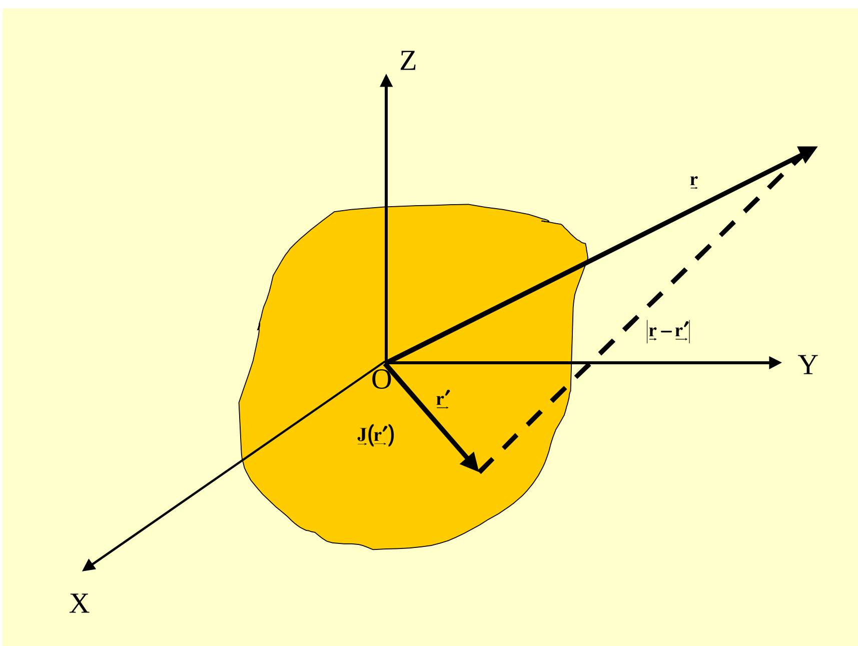

Figure (A.1). The general coordinates where $\underset{\rightarrow}{r}$ is a field point and $\underset{\rightarrow}{r}'$ is a source point.

Consider a system whose charges and currents are varying in time. There is no loss in generality by restricting our considerations to potentials, field and radiation from localized systems that vary sinusoidally in time.

The time dependent charge and current densities could be written, respectively, as given by equation (**A.1**) and equation (**A.2**), based on coordinates shown in figure (**A.1**).

$$\rho(\underline{\vec{r}}',t) = \rho(\underline{\vec{r}})\,e^{-i\omega t} \tag{A.1}$$

$$J(\underline{\vec{r}',t}) = J(\underline{\vec{r}})\,e^{-i\omega t} \tag{A.2}$$

And the continuity equation is given by equation (**A.3**).

$$\nabla.J + \frac{\partial\rho}{\partial t} = \nabla.J + (-i\omega\rho) = 0 \tag{A.3}$$

The scalar and vector potentials for the charge distributions are given, respectively, by equation (**A.4**) and equation (**A.5**).

$$V(\underline{\vec{r}}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\underline{\vec{r}}')\,e^{ik|\underline{\vec{r}}-\underline{\vec{r}}'|}}{|\underline{\vec{r}}-\underline{\vec{r}}'|}\,d^3\underline{\vec{r}}' \tag{A.4}$$

$$\underline{\vec{A}}(\underline{\vec{r}}) = \frac{\mu_0}{4\pi}\int \frac{\underline{\vec{J}}(\underline{\vec{r}}')\,e^{ik|\underline{\vec{r}}-\underline{\vec{r}}'|}}{|\underline{\vec{r}}-\underline{\vec{r}}'|}\,d^3\underline{\vec{r}}' \tag{A.5}$$

In the far radiation zone, where **r** >> **λ** and **r** >> **r$^{/}$,** the denominator |**r**- **r**$^{/}$| can be considered to be independent of **r**$^{/}$ although the argument of the complex exponential is not. Thus, equation **(A.5)** could be written as equation (**A.6**).

$$\underline{\mathbf{A}}(\underline{\mathbf{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \underline{\mathbf{J}}(\underline{\mathbf{r}}')e^{-ik(\underline{\mathbf{r}}.\underline{\mathbf{r}}'/r)}\, d^3\underline{\mathbf{r}}' \tag{A.6}$$

Also if the source dimension r$^{/}$ is small compared to the wavelength, then k r$^{/}$ << 1 which justifies writing equation **(A.6)** as equation (**A.7**).

$$\underline{\mathbf{A}}(\underline{\mathbf{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\sum_l \frac{(-ik)^l}{l!}\int \underline{\mathbf{J}}(\underline{\mathbf{r}}')\left(\frac{\underline{\mathbf{r}}.\underline{\mathbf{r}}'}{r}\right)^l d^3\underline{\mathbf{r}}' \tag{A.7}$$

The above is true since the electron diameter of 5×10^{-15} m is very small compared to the wavelength of visible light of 5×10^{-07} m. Therefore the expansion of the exponential term in the integrand of equation (**A.6**) is justified by the summation given equation (**A.8**).

$$e^{-ik\left(\underline{\mathbf{r}}.\underline{\mathbf{r}}'/r\right)} = \sum_l \frac{(-ik)^l}{l!}\left(\underline{\mathbf{r}}.\underline{\mathbf{r}}'/r\right)^l \tag{A.8}$$

The scattering of electromagnetic radiation by systems whose physical dimensions are small compared with the wavelength of the wave being scattered, it is reasonable to assume that the incident radiation as inducing electric and magnetic multipoles and these to oscillate in definite phase relationship with the incident wave and also radiate energy in directions different from the direction of the incident wave. For the case where the wavelength of the incident radiation is very long compared to the size of the scatterer, only the lowest multipoles, usually the electric and magnetic dipoles (**l=0**) are important.Therefore for electric dipole radiation (**l=0**) equation (**A.7**) reduces to equation (**A.9**).

$$\underset{\rightarrow}{\underline{A}}(\underset{\rightarrow}{\underline{r}}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \underset{\rightarrow}{\underline{J}}(\underset{\rightarrow}{\underline{r'}})\,d^3\underset{\rightarrow}{\underline{r'}} \qquad \textbf{(A.9)}$$

Consider the following mathematical operation

$$\begin{aligned}
&\nabla'.(x'\underset{\rightarrow}{\underline{J}}) = (\nabla'x').J + x'(\nabla'.\underset{\rightarrow}{\underline{J}}) \\
&\qquad = J_x + x'(\nabla'.\underset{\rightarrow}{\underline{J}}) \\
&\therefore J_x = \nabla'.(x'\underset{\rightarrow}{\underline{J}}) - x'(\nabla'.\underset{\rightarrow}{\underline{J}}) \\
&\int J_x d^3\underset{\rightarrow}{\underline{r}}' = \int \nabla'.(x'\underset{\rightarrow}{\underline{J}})d^3\underset{\rightarrow}{\underline{r}}' - \int x'(\nabla'.\underset{\rightarrow}{\underline{J}})d^3\underset{\rightarrow}{\underline{r}}' \\
&\qquad = \oint x'\underset{\rightarrow}{\underline{J}}.ds - \int x'(\nabla'.\underset{\rightarrow}{\underline{J}})d^3\underset{\rightarrow}{\underline{r}}' \\
&\textbf{Using equation(1.3)} \\
&\qquad = \oint x'\underset{\rightarrow}{\underline{J}}.d\underset{\rightarrow}{\underline{s}} + \int x'\frac{\partial\rho}{\partial t}d^3\underset{\rightarrow}{\underline{r}}' \\
&\textbf{Using equation(1.1) and integrating over a large enough volume} \\
&\qquad \cong -i\omega\int x'\rho\, d^3\underset{\rightarrow}{\underline{r}}' \\
&\textbf{and for all components of } \underset{\rightarrow}{\underline{J}} \\
&\int \underset{\rightarrow}{\underline{J}}(\underset{\rightarrow}{\underline{r}}')\,d^3\underset{\rightarrow}{\underline{r}}' = -i\omega\int r'\rho(\underset{\rightarrow}{\underline{r}}')\,d^3\underset{\rightarrow}{\underline{r}}' = -i\omega\underset{\rightarrow}{\underline{p}} \qquad \textbf{(A.10)} \\
&\textbf{where dipole moment } \underset{\rightarrow}{\underline{p}} = \int r'\rho(\underset{\rightarrow}{\underline{r}}')\,d^3\underset{\rightarrow}{\underline{r}}'
\end{aligned}$$

Equation (**A.11**) can be written from equation (**A.9**) and equation (**A.10**).

$$\underset{\rightarrow}{\underline{A}}(\underset{\rightarrow}{\underline{r}}) = \frac{-i\omega\mu_0}{4\pi}\frac{e^{ikr}}{r}\underset{\rightarrow}{\underline{p}} \tag{A.11}$$

Then, using equation (**A.11**) the magnetic field is given by equation (**A.12**).

$$\boxed{\begin{array}{l} \underset{\rightarrow}{\underline{B}} = \nabla \times \underset{\rightarrow}{\underline{A}}(\underset{\rightarrow}{\underline{r}}) = -\dfrac{i\omega\mu_o}{4\pi}\nabla\left(\dfrac{e^{ikr}}{r} \times \underset{\rightarrow}{\underline{p}}\right) \\ \textbf{and using polar coordinates} \\ \nabla(r,\theta,\phi) = \underset{\rightarrow}{\hat{r}}\dfrac{\partial}{\partial r} + \dfrac{\underset{\rightarrow}{\hat{\theta}}}{r}\dfrac{\partial}{\partial \theta} + \dfrac{\underset{\rightarrow}{\hat{\phi}}}{r\sin(\theta)}\dfrac{\partial}{\partial \phi} \\ \underset{\rightarrow}{\underline{B}} = \dfrac{\omega k\mu_o}{4\pi}\left(1 - \dfrac{1}{ikr}\right)\dfrac{e^{ikr}}{r}\underset{\rightarrow}{\hat{r}} \times \underset{\rightarrow}{\underline{p}} \end{array}} \tag{A.12}$$

From Maxwell's equations for outside the source (**$\underline{J}$**=0) the magnetic and the electric fields are related by equation (**A.13**).

$$\boxed{\begin{array}{l} \nabla \times \underset{\rightarrow}{\underline{B}} = \mu_o\varepsilon_o\dfrac{\partial \underset{\rightarrow}{\underline{E}}}{\partial t} + \mu\underset{\rightarrow}{\underline{J}} = -\dfrac{i\omega}{c^2}\underset{\rightarrow}{\underline{E}} \\ \text{where } c^2 = \dfrac{1}{\mu_o\varepsilon_o} \text{ and } k = \dfrac{\omega}{c} \end{array}} \tag{A.13}$$

From equation (**A.13**) the electric field is given by equation (**A.14**).

$$\underset{\rightarrow}{\underline{E}}(\underset{\rightarrow}{\underline{r}}) = \frac{ic^2}{\omega}\nabla \times \underset{\rightarrow}{\underline{B}}(\underset{\rightarrow}{\underline{r}}) \tag{A.14}$$

And using the following vector identities

$$\underline{\vec{A}}\times\left(\underline{\vec{B}}\times\underline{\vec{C}}\right)=\left(\underline{\vec{A}}.\underline{\vec{C}}\right)\underline{\vec{B}}-\left(\underline{\vec{A}}.\underline{\vec{B}}\right)\underline{\vec{C}}$$
$$\nabla\times\left(\underline{\vec{A}}\times\underline{\vec{B}}\right)=\underline{\vec{A}}\left(\nabla.\underline{\vec{B}}\right)-\underline{\vec{B}}\left(\nabla\times\underline{\vec{A}}\right)+\left(\underline{\vec{B}}.\nabla\right)\underline{\vec{A}}-\left(\underline{\vec{A}}.\nabla\right)\underline{\vec{B}}$$

and the relations

$$\nabla.\underline{\vec{\hat{r}}}=\frac{2}{r},\ \nabla.\underline{\vec{r}}=3\ \text{and}\ \underline{\vec{p}}\neq\underline{\vec{p}}\left(\underline{\vec{r}}\right)$$

on equation (**A.14**) the expression for the electric field is given by equation (**A.15**).

$$\underline{\vec{E}}\left(\underline{\vec{r}}\right)=\frac{ck\omega\mu_o}{4\pi}\frac{e^{ikr}}{r}\underline{\vec{\hat{r}}}\times\left(\underline{\vec{p}}\times\underline{\vec{\hat{r}}}\right)+\frac{1}{4\pi\varepsilon_o}\frac{\left(1-ikr\right)}{r^3}e^{ikr}\left(3\left(\underline{\vec{\hat{r}}}.\underline{\vec{p}}\right)-\underline{\vec{p}}\right) \qquad \textbf{(A.15)}$$

In the radiation zone **kr>>1** where $k=\omega/c$ the equations (**A.15**) and (**A.12**) reduce to equation (**A.16**).

$$\boxed{\begin{array}{l}\underline{\vec{E}}\left(\underline{\vec{r}}\right)\cong\dfrac{ck\omega\mu_o}{4\pi}\dfrac{e^{ikr}}{r}\underline{\vec{\hat{r}}}\times\left(\underline{\vec{p}}\times\underline{\vec{\hat{r}}}\right)\\ \underline{\vec{B}}\left(\underline{\vec{r}}\right)\cong\dfrac{k\omega\mu_o}{4\pi}\dfrac{e^{ikr}}{r}\underline{\vec{\hat{r}}}\times\underline{\vec{p}}\\ \underline{\vec{E}}\left(\underline{\vec{r}}\right)=c\underline{\vec{B}}\left(\underline{\vec{r}}\right)\times\underline{\vec{\hat{r}}}\end{array}} \qquad \textbf{(A.16)}$$

The Poynting vector (average power radiated per unit area) is given by equation (**A.17**).

$$\langle \underset{\rightarrow}{s} \rangle = \frac{\underset{\rightarrow}{E} \times \underset{\rightarrow}{B}^*}{2\mu_o}\ \mathbf{Watts/m^2} \qquad \textbf{(A.17)}$$

Using equation (**A.16**) and the vector identity

$(\underset{\rightarrow}{A} \times \underset{\rightarrow}{B}) \times \underset{\rightarrow}{C} = (\underset{\rightarrow}{A}.\underset{\rightarrow}{C})\underset{\rightarrow}{B} - (\underset{\rightarrow}{B}.\underset{\rightarrow}{C})\underset{\rightarrow}{A}$ **and** $\underset{\rightarrow}{D}.(\underset{\rightarrow}{D} \times \underset{\rightarrow}{E}) = 0$ and $\mathbf{k = \omega / c}$ the Poynting vector given by equation (**A.17**) reduces to equation (**A.18**).

$$\langle \underset{\rightarrow}{s} \rangle = \frac{\omega^4 \mu_o}{32\pi^2 r^2 c} (\underset{\rightarrow}{\hat{r}} \times \underset{\rightarrow}{p})^2 \underset{\rightarrow}{\hat{r}} \quad \mathbf{Watts/m^2} \qquad \textbf{(A.18)}$$

Then the total power radiated through an area $\mathbf{d\underset{\rightarrow}{a}}$ is given by equation (**A.19**).

$$\mathbf{dL} = \langle \underset{\rightarrow}{S} \rangle . d\underset{\rightarrow}{a} = \langle \underset{\rightarrow}{S} \rangle . \underset{\rightarrow}{\hat{r}} r^2 d\Omega \quad \mathbf{Watts} \qquad \textbf{(A.19)}$$

Substituting equation (**A.18**) in (**A.19**) gives equation (**A.20**).

$$\mathbf{dL} = \frac{\omega^4 \mu_o}{32\pi^2 r^2 c} (\underset{\rightarrow}{\hat{r}} \times \underset{\rightarrow}{p})^2 \underset{\rightarrow}{\hat{r}} . \underset{\rightarrow}{\hat{r}} r^2 d\Omega = \frac{\omega^4 \mu_o}{32\pi^2 c} (\underset{\rightarrow}{\hat{r}} \times \underset{\rightarrow}{p})^2 d\Omega \quad \mathbf{Watts} \qquad \textbf{(A.20)}$$

Now for an oscillating electric dipole the induced moment $\underset{\rightarrow}{\mathbf{p}}$ is given by equation (**A.21**).

$$\underset{\rightarrow}{p} = \underset{\rightarrow}{p}_o e^{-i\omega t} \qquad \textbf{(A.21)}$$

Differentiating equation (**A.21**) twice with respect to time gives equation (**A.22**).

$$\ddot{\underset{\rightarrow}{\mathbf{p}}} = \omega^2 \underset{\rightarrow}{\mathbf{p}} \tag{A.22}$$

Substituting equation (**A.22**) in (**A.20**) gives equation (**A.23**).

$$\mathbf{dL} = \frac{\mu_o}{32\pi^2 c}\left(\hat{\underset{\rightarrow}{\mathbf{r}}} \times \ddot{\underset{\rightarrow}{\mathbf{p}}}\right)^2 \mathbf{d\Omega} \quad \textbf{Watts} \tag{A.23}$$

For a single electron the induced moment $\underset{\rightarrow}{\mathbf{p}}$ is given by

$$\underset{\rightarrow}{\mathbf{p}}(\mathbf{t}) = \mathbf{e}\underset{\rightarrow}{\mathbf{r}}'(\mathbf{t}) \tag{A.24}$$

where $\underset{\rightarrow}{\mathbf{r}}'$ is a source point with respect to its origin.

From the equation of motion in the non-relativistic case

$$\mathbf{m}\underset{\rightarrow}{\mathbf{r}}' = \mathbf{e}\left(\underset{\rightarrow}{\mathbf{E}} + \underset{\rightarrow}{\mathbf{v}} \times \underset{\rightarrow}{\mathbf{B}}\right) \cong \mathbf{e}\underset{\rightarrow}{\mathbf{E}} \tag{A.25}$$

where **m** is the mass of the electron.

Substituting equation (**A.25**) in (**A.24**) gives equation (**A.26**).

$$\ddot{\underset{\rightarrow}{\mathbf{p}}} = \frac{\mathbf{e}^2 \underset{\rightarrow}{\mathbf{E}}}{\mathbf{m}} \tag{A.26}$$

Again substituting equation (**2.26**) in (**2.23**) gives equation (**A.27**).

$$dL = \left(\frac{e^2}{mc^2}\right)^2 \frac{\mu_0 c^3}{32\pi^2}\left(\hat{\underset{\rightarrow}{r}} \times \underset{\rightarrow}{E}\right)^2 d\Omega \qquad \textbf{(A.27)}$$

Diving and multiplying equation (**A.27**) by $\mathbf{6\pi\varepsilon_o}$ gives equation (**A.28**).

$$dL = \frac{3}{16\pi}\sigma_T \varepsilon_0 c\left(\hat{\underset{\rightarrow}{r}} \times \underset{\rightarrow}{E}\right)^2 d\Omega \qquad \textbf{(A.28)}$$

where $\boldsymbol{\sigma}_T$ is the Thomson scattering cross section given by equation (**A.29**).

$$\sigma_T = \frac{1}{6\pi\varepsilon_0}\left(\frac{e^2}{mc^2}\right)^2 \qquad \textbf{(A.29)}$$

Therefore from equation (**A.28**) the power radiated per unit solid angle is given by equation (**A.30**).

$$\boxed{\frac{dL}{d\Omega} = \frac{3}{16\pi}\sigma_T\left(\hat{\underset{\rightarrow}{r}} \times \underset{\rightarrow}{E}\right)^2 \ \textbf{Watts/solid angle}} \qquad \textbf{(A.30)}$$

A.2 Total power radiated per solid angle for the cases where the electric vector is aligned parallel and perpendicular to the scattering plane

Consider the case where the electric vector is parallel to the scattering plane. In figure (**A.2**) $\underline{\mathbf{n}}_{\mathbf{in}}$ and $\underline{\mathbf{n}}_{\mathbf{out}}$ are unit vectors in the directions of the incident and the scattered radiation and they both lie on the scattering plane and $\boldsymbol{\Theta}$ is the scattering angle. $\hat{\underline{\mathbf{E}}}_{//}^{\mathbf{Sc}}$ is the unit vector of the component of the electric vector parallel to the scattering plane and $\hat{\underline{\mathbf{E}}}_{\perp}^{\mathbf{Sc}}$ is the unit vector of the component of the electric vector perpendicular to the scattering plane.

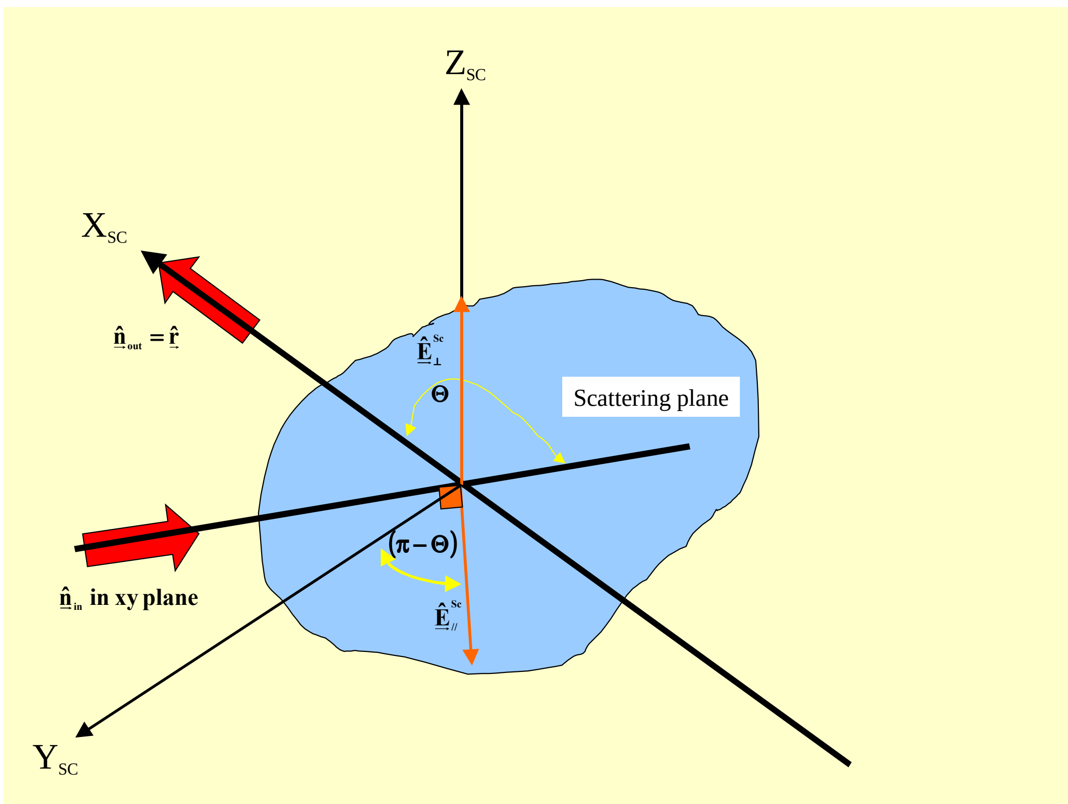

Figure (A.2). The general coordinates of the incident and scattered radiation and the orientations of the parallel and the perpendicular components of the electric vectors with respect to the scattering plane.

The direction cosines of $\hat{\underline{\mathbf{n}}}_{in}$ and $\hat{\underline{\mathbf{n}}}_{out}$ are given by equation (**A.31**) and equation (**A.32**), respectively.

$$\hat{\underline{\mathbf{n}}}_{\mathbf{in}} = \left(-\sin\left(\Theta - \frac{\pi}{2}\right), -\cos\left(\Theta - \frac{\pi}{2}\right), 0\right) = (\cos\Theta, -\sin\Theta, 0) \tag{A.31}$$

$$\hat{\underline{\mathbf{n}}}_{\mathbf{out}} = (1,0,0) \tag{A.32}$$

From figure (**A.2**) the dot product of $\hat{\underline{\mathbf{n}}}_{in}$ and $\hat{\underline{\mathbf{n}}}_{out}$ gives equation (**A.33**)

$$\hat{\underline{\mathbf{n}}}_{\mathbf{in}} \cdot \hat{\underline{\mathbf{n}}}_{\mathbf{out}} = |\hat{\underline{\mathbf{n}}}_{\mathbf{in}}||\hat{\underline{\mathbf{n}}}_{\mathbf{out}}|\cos(\Theta) \tag{A.33}$$

and the square of its cross product gives equation (**A.34**).

$$(\hat{\underline{\mathbf{n}}}_{\mathbf{in}} \times \hat{\underline{\mathbf{n}}}_{\mathbf{out}})^2 = |\hat{\underline{\mathbf{n}}}_{\mathbf{in}}|^2 |\hat{\underline{\mathbf{n}}}_{\mathbf{out}}|^2 \sin^2(\Theta) \tag{A.34}$$

The direction cosines for the electric vectors $\hat{\underline{\mathbf{E}}}_{//}^{\mathbf{Sc}}$ and $\hat{\underline{\mathbf{E}}}_{\perp}^{\mathbf{Sc}}$ are, respectively, given by equation (**A.35**) and equation (**A.36**).

$$\hat{\underline{\mathbf{E}}}_{//}^{\mathbf{Sc}} = (-\sin(\pi - \Theta), \cos(\pi - \Theta), 0) = (-\sin\Theta, -\cos\Theta, 0) \tag{A.35}$$

$$\hat{\underline{\mathbf{E}}}_{\perp}^{\mathbf{Sc}} = (0,0,1) \tag{A.36}$$

The sum of the cross products of equation (**A.32**) with (**A.34**) and (**A.35**) is gives equation (**A.37**).

$$\hat{\underset{\rightarrow}{\mathbf{n}}}_{\mathbf{out}} \times \hat{\underset{\rightarrow}{\mathbf{E}}}_{//}^{\mathbf{Sc}} + \hat{\underset{\rightarrow}{\mathbf{n}}}_{\mathbf{out}} \times \hat{\underset{\rightarrow}{\mathbf{E}}}_{\perp}^{\mathbf{Sc}} = \begin{vmatrix} \hat{\underset{\rightarrow}{\mathbf{i}}} & \hat{\underset{\rightarrow}{\mathbf{j}}} & \hat{\underset{\rightarrow}{\mathbf{k}}} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \\ -\sin\Theta & -\cos\Theta & \mathbf{0} \end{vmatrix} + \begin{vmatrix} \hat{\underset{\rightarrow}{\mathbf{i}}} & \hat{\underset{\rightarrow}{\mathbf{j}}} & \hat{\underset{\rightarrow}{\mathbf{k}}} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{vmatrix} \qquad \textbf{(A.37)}$$

$$= -\cos\Theta \hat{\underset{\rightarrow}{\mathbf{k}}} + \hat{\underset{\rightarrow}{\mathbf{j}}}$$

From figure (**A.2**) using the relation $\hat{\underset{\rightarrow}{\mathbf{r}}} = \hat{\underset{\rightarrow}{\mathbf{n}}}_{\mathbf{out}}$ and equation (**A.37**) the following relations are obtained as given by equation (**A.38**) and equation (**A.39**).

$$\left(\hat{\underset{\rightarrow}{\mathbf{r}}} \times \hat{\underset{\rightarrow}{\mathbf{E}}}_{//}^{\mathbf{Sc}} \mathbf{E}_{//}^{\mathbf{Sc}}\right)^{2} = (\mathbf{E}_{//}^{\mathbf{Sc}})^{2} \sin^{2}(90 - \Theta) = (\mathbf{E}_{//}^{\mathbf{Sc}})^{2} \cos^{2}\Theta \qquad \textbf{(A.38)}$$

$$\left(\hat{\underset{\rightarrow}{\mathbf{r}}} \times \hat{\underset{\rightarrow}{\mathbf{E}}}_{\perp}^{\mathbf{Sc}} \mathbf{E}_{\perp}^{\mathbf{Sc}}\right)^{2} = (\mathbf{E}_{\perp}^{\mathbf{Sc}})^{2} \qquad \textbf{(A.39)}$$

Now substituting the equation (**A.38**) and equation (**A.39**) in equation (**A.30**) give equation (**A.40**) and equation (**A.41**), respectively.

$$\left(\frac{\mathbf{dL}}{\mathbf{d\Omega}}\right)_{//}^{\mathbf{Sc}} = \frac{\mathbf{3}}{\mathbf{16\pi}} \sigma_{\mathbf{T}} \left(\varepsilon_{\mathbf{0}} \mathbf{c} (\mathbf{E}_{//}^{\mathbf{Sc}})^{2}\right) \cos^{2}\Theta \qquad \textbf{(A.40)}$$

$$\left(\frac{\mathbf{dL}}{\mathbf{d\Omega}}\right)_{\perp}^{\mathbf{Sc}} = \frac{\mathbf{3}}{\mathbf{16\pi}} \sigma_{\mathbf{T}} \left(\varepsilon_{\mathbf{0}} \mathbf{c} (\mathbf{E}_{\perp}^{\mathbf{Sc}})^{2}\right) \qquad \textbf{(A.41)}$$

But for isotropic radiation equation (**A.42**) is true.

$$\left|\underset{\rightarrow}{\mathbf{E}}_{//}^{\mathbf{Sc}}\right| = \left|\underset{\rightarrow}{\mathbf{E}}_{\perp}^{\mathbf{Sc}}\right| = \frac{1}{2}\left|\underset{\rightarrow}{\mathbf{E}}\right| \tag{A.42}$$

Substituting equation (**A.42**) in (**A.40**) and (**A.41**) give equation (**A.43**) and (**A.44**), respectively.

$$\left(\frac{\mathbf{dL}}{\mathbf{d\Omega}}\right)_{//}^{\mathbf{Sc}} = \frac{3}{16\pi}\sigma_T\left(\frac{1}{2}c\varepsilon_0 E^2\right)\cos^2\Theta \tag{A.43}$$

$$\left(\frac{\mathbf{dL}}{\mathbf{d\Omega}}\right)_{\perp}^{\mathbf{Sc}} = \frac{3}{16\pi}\sigma_T\left(\frac{1}{2}c\varepsilon_0 E^2\right) \tag{A.44}$$

Consider a plane monochromatic wave as shown in figure (**A.3**).

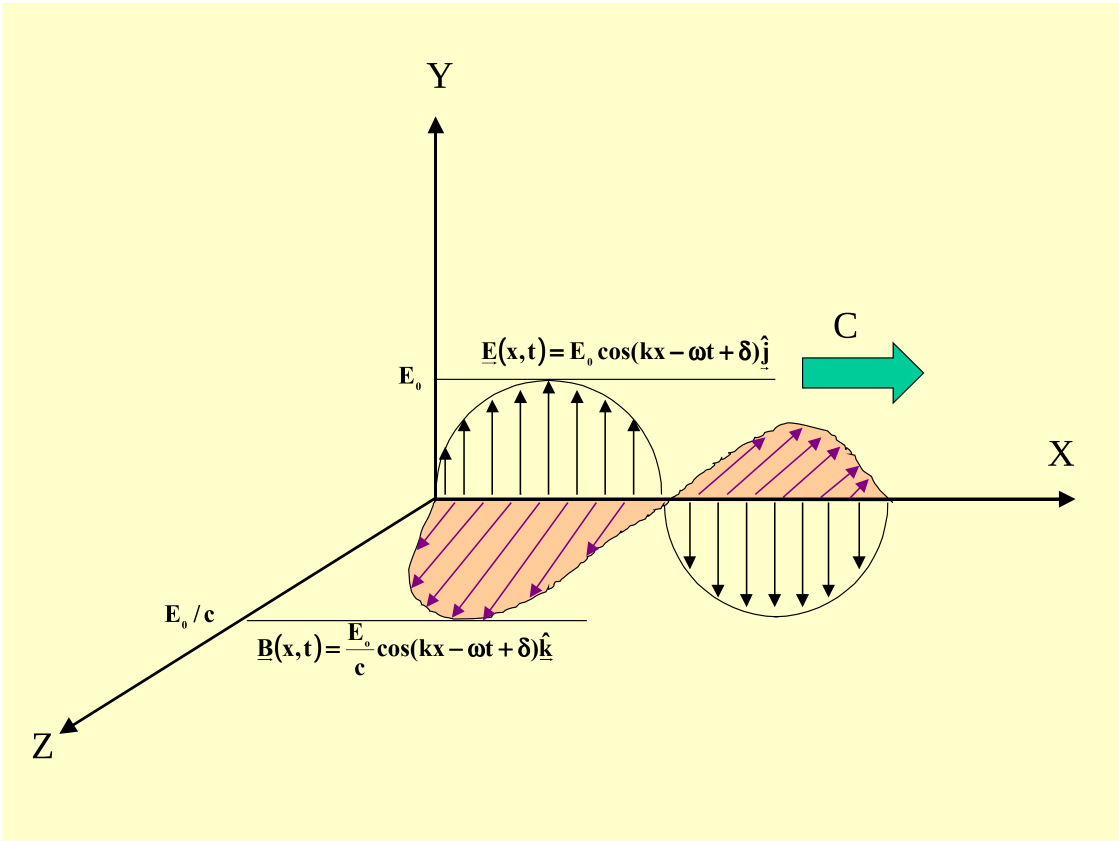

Figure (A.3). Propagation of a monochromatic plane EM waves.

Using the Poynting vector $\underset{\rightarrow}{\mathbf{S}} = \frac{1}{\mu_o}\underset{\rightarrow}{\mathbf{E}} \times \underset{\rightarrow}{\mathbf{B}}$ the energy flux density transported by the plane monochromatic electromagnetic wave depicted in figure (**A.3**) is given by equation (**A.45**).

$$\underset{\rightarrow}{\mathbf{S}} = c\varepsilon_0 E_0^2 \cos^2(kx - \omega t + \delta)\underset{\rightarrow}{\hat{\mathbf{i}}} \qquad \textbf{(A.45)}$$

and averaging equation (**A.45**) over a cycle gives equation (**A.46**).

$$\boxed{\begin{aligned}\langle \underset{\rightarrow}{S} \rangle &= c\varepsilon_0 E_o^2 \left\langle \cos^2\left(kx - \frac{2\pi}{T}t + \delta\right)\right\rangle \underset{\rightarrow}{\hat{i}} \\ &= c\varepsilon_0 E_o^2 \frac{1}{T}\int_0^T \cos^2\left(kx - \frac{2\pi}{T}t + \delta\right) dt\, \underset{\rightarrow}{\hat{i}} \\ &= \frac{1}{2} c\varepsilon_0 E_o^2 \underset{\rightarrow}{\hat{i}}\end{aligned}} \qquad \text{(A.46)}$$

From equation (**A.46**) the average power per unit area transported by an electromagnetic wave is called the intensity and is given by equation (**A.47**).

$$I_o = \langle S \rangle = \frac{1}{2} c\varepsilon_o E_o^2 \qquad \text{(A.47)}$$

Substituting equation (**A.47**) in equation (**A.43**) and equation (**A.44**) give equation (**A.48**) and equation (**A.49**), respectively.

$$I_{//}^{Sc} \equiv \left(\frac{dL}{d\Omega}\right)_{//}^{Sc} = \frac{3}{16\pi}\sigma_T I_o \cos^2\Theta \quad \text{Joules/sec.steradian.}\overset{0}{\text{A}}\text{.electron} \qquad \text{(A.48)}$$

$$I_{\perp}^{Sc} \equiv \left(\frac{dL}{d\Omega}\right)_{\perp}^{Sc} = \frac{3}{16\pi}\sigma_T I_o \quad \text{Joules/sec.steradian.}\overset{0}{\text{A}}\text{.electron} \qquad \text{(A.49)}$$

where I_o is the incident radiation.

Consider a radial plane inclined at an angle α to the scattering plane, as shown in figure (**A.4**). The variation of intensity through an angle β is given by equation (**A.50**).

$$\mathbf{I = I_M \cos^2 \beta} \qquad \textbf{(A.50)}$$

where $\mathbf{I_M}$ is the maximum intensity.

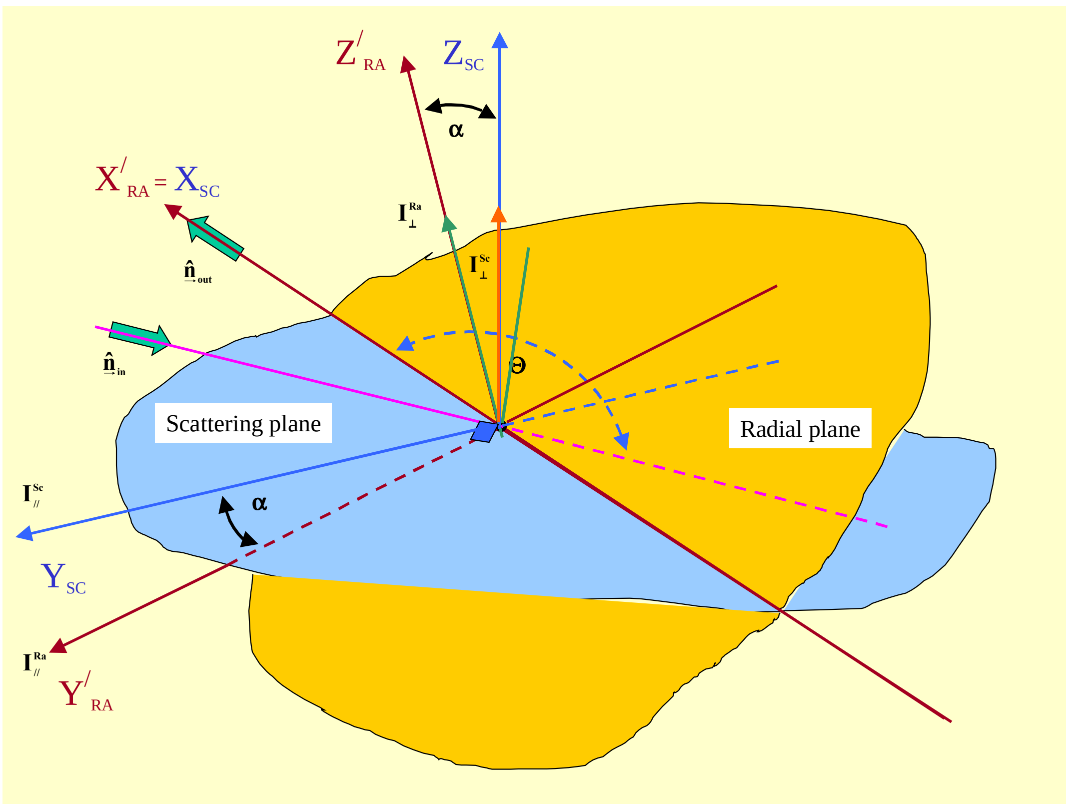

Figure (A.4). Coordinates for the rotational transformation of the scattering plane through an angle α about the line of sight.

From figure (**A.4**) and using equation (**A.50**) equation (**A.51**) and equation (**A.52**) are obtained for the intensity parallel and perpendicular to the radial plane, respectively.

$$\begin{aligned} \mathbf{I}_{//}^{\mathbf{Ra}} &= \mathbf{I}_{//}^{\mathbf{Sc}} \cos^2 \alpha + \mathbf{I}_{\perp}^{\mathbf{Sc}} \cos^2\left(\frac{\pi}{2} - \alpha\right) \\ &= \mathbf{I}_{//}^{\mathbf{Sc}} \cos^2 \alpha + \mathbf{I}_{\perp}^{\mathbf{Sc}} \sin^2 \alpha \end{aligned} \tag{A.51}$$

$$\begin{aligned} \mathbf{I}_{\perp}^{\mathbf{Ra}} &= \mathbf{I}_{\perp}^{\mathbf{Sc}} \cos^2 \alpha + \mathbf{I}_{//}^{\mathbf{Sc}} \cos^2\left(\frac{\pi}{2} - \alpha\right) \\ &= \mathbf{I}_{//}^{\mathbf{Sc}} \sin^2 \alpha + \mathbf{I}_{\perp}^{\mathbf{Sc}} \cos^2 \alpha \end{aligned} \tag{A.52}$$

Substituting equation (**A.48**) and equation (**A.49**) in equation (**A.51**) and equation (**A.52**) give equation (**A.53**) and equation (**A.54**), respectively.

$$\boxed{\begin{aligned} \mathbf{I}_{//}^{\mathbf{Ra}} \equiv \left(\frac{\mathbf{dL}}{\mathbf{d\Omega}}\right)_{//}^{\mathbf{Ra}} &= \frac{3}{16\pi}\sigma_T \mathbf{I}_0\left(\sin^2 \alpha + \cos^2 \Theta \cos^2 \alpha\right) \\ &= \mathbf{I}_{\perp}^{\mathbf{Sc}} \sin^2 \alpha + \mathbf{I}_{//}^{\mathbf{Sc}} \cos^2 \alpha \\ &\equiv \mathbf{Q}_{//}^{\mathbf{Ra}}(\alpha, \Theta)\mathbf{I}_o \ \textbf{Joules/sec.steradian.A}^{o}\textbf{.electron} \end{aligned}} \tag{A.53}$$

$$\boxed{\begin{aligned} \mathbf{I}_{\perp}^{\mathbf{Ra}} \equiv \left(\frac{\mathbf{dL}}{\mathbf{d\Omega}}\right)_{\perp}^{\mathbf{Ra}} &= \frac{3}{16\pi}\sigma_T \mathbf{I}_0\left(\cos^2 \alpha + \cos^2 \Theta \sin^2 \alpha\right) \\ &= \mathbf{I}_{\perp}^{\mathbf{Sc}} \cos^2 \alpha + \mathbf{I}_{//}^{\mathbf{Sc}} \sin^2 \alpha \\ &\equiv \mathbf{Q}_{\perp}^{\mathbf{Ra}}(\alpha, \Theta)\mathbf{I}_o \ \textbf{Joules/sec.steradian.A}^{o}\textbf{.electron} \end{aligned}} \tag{A.54}$$

Using the scattering theories derived above the same could be applied for the scattering of photospheric radiation by the coronal electrons. Figure (**A.5**) shows the corresponding geometrical configurations depicted by figures (**A.2**) and (**A.4**) for the case of photospheric radiation being scattered by the coronal electrons to an observer on earth.

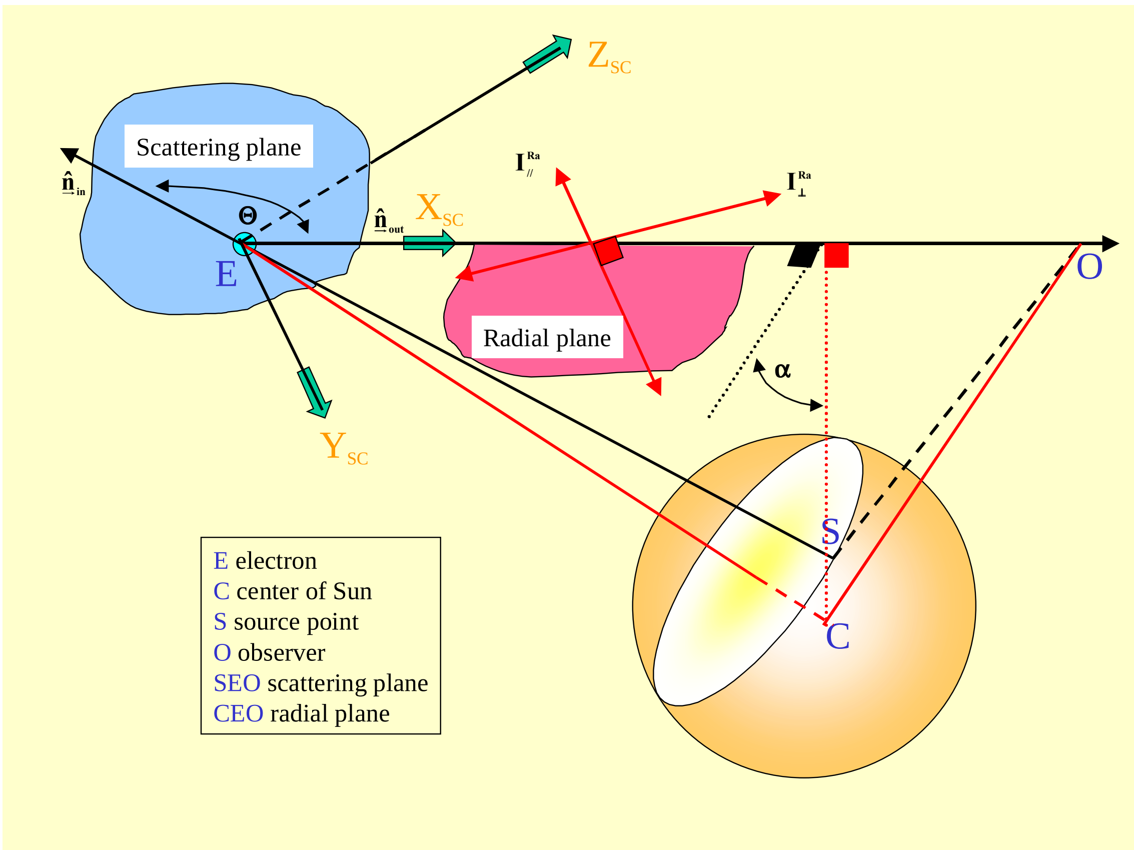

Figure (A.5). Geometrical configuration for the scattering of photospheric radiation by coronal electrons.

The following figure (**A.6**) shows the geometrical configuration for the formation of the K-Corona, which also incorporates the effect due to the radial solar wind.

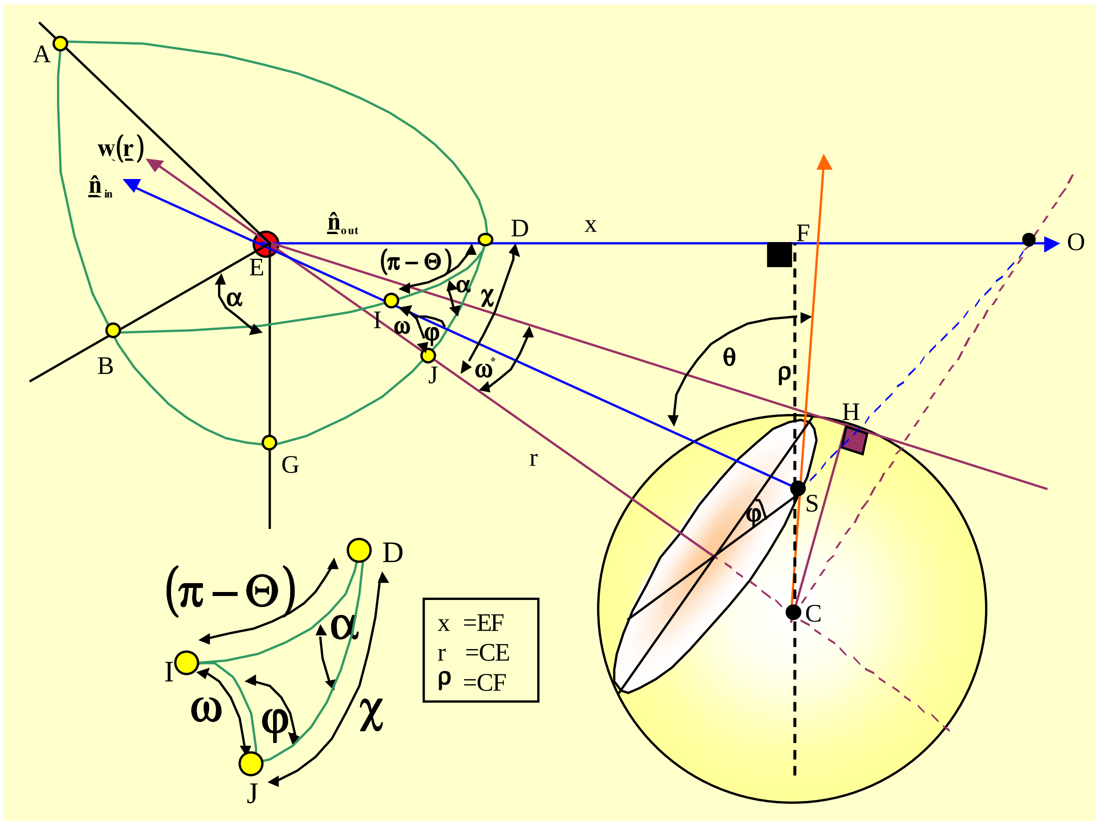

Figure (A.6). Geometrical configuration for the formation of the K- Corona. Photospheric radiation emitted from a point S on the Sun is scattered from an electron E towards an observer O. The solar wind on the electron is radial and blows in the direction CE.

A.3 Contribution due to Compton scattering

In the rest frame of the electron the scattering of the photospheric light incident on the electron is coherent. The red shift due to the Compton effect is very negligible. To quantify the contribution due to Compton effect, consider radiation of wavelength λ incident on a stationary electron, as shown in figure (**A.7**). Let the radiation scattered off the electron be of wavelength λ' and the velocity of the electron now be $\underset{\rightarrow}{\mathbf{v}}$. The scattering angles for the radiation and the electron are, θ and φ, respectively.

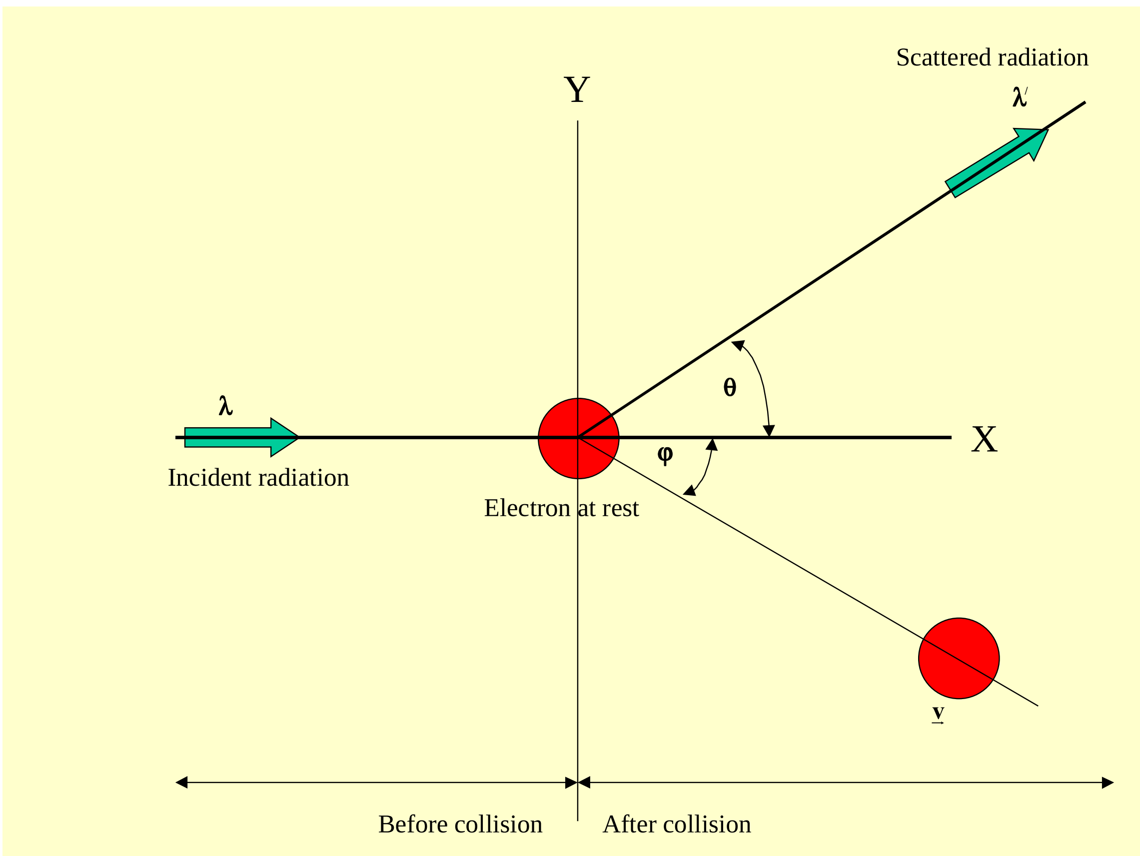

Figure (A.7). Scattering of radiation off an electron also known as Compton scattering.

From conservation of energy in figure (**A.7**) equation (**A.55**) is obtained.

$$\frac{hc}{\lambda}=\frac{hc}{\lambda'}+mc^2\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}-1\right) \tag{A.55}$$

From figure (**A.7**) using conservation of momentum in the X the Y directions, respectively, equation (**A.56**) and equation (**A.57**) are obtained.

$$\frac{h}{\lambda}=\frac{h}{\lambda'}\cos\theta+\frac{mv}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\cos\varphi \tag{A.56}$$

$$0=\frac{h}{\lambda'}\sin\theta+\frac{mv}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\sin\varphi \tag{A.57}$$

where

$$\begin{aligned}
&\mathbf{h = Planck\ constant = 6.63\times10^{-34}\ J.s}\\
&\mathbf{m = electron\ mass = 9.11\times10^{-31}\ Kg}\\
&\mathbf{c = speed\ of\ light = 3\times10^{8}\ ms^{-1}}
\end{aligned}$$

Eliminating φ and **v** from equation (**A.55**), (**A.56**) and (**A.57**), the wavelength shift is given by equation (**A.58**).

$$\begin{aligned}\Delta\lambda=\lambda'-\lambda&=\frac{h}{mc}(1-\cos\theta)\\&=0.0243A^0(1-\cos\theta)\le0.0486A^0\end{aligned} \tag{A.58}$$

A.4 Expression for the total K-Coronal scattered intensity

From figure (**A.6**) for the calculation of the total K-Coronal scattered intensity in the rest frame of the electron, first, it is necessary to consider an electron velocity distribution over a volume element at P in order to determine the number density. Consider a Maxwellian velocity distribution for the coronal electrons. Then the number density at the point P in the velocity interval $\left(\underset{\rightarrow}{\mathbf{u}},\underset{\rightarrow}{\mathbf{u}}+\mathbf{d}\underset{\rightarrow}{\mathbf{u}}\right)$ is given by equation (**A.59**).

$$\boxed{\begin{array}{l} \mathbf{f_e\left(\underset{\rightarrow}{u}\right)=N_e\left(P\right)\frac{1}{\left(\sqrt{\pi q}\right)^3}e^{\left(-\frac{u^2}{q^2}\right)}} \quad \textbf{where} \\ \mathbf{q\left(P\right)=mean\ electron\ thermal\ velocity}=\sqrt{\frac{\mathbf{2kT_e}}{\mathbf{m_e}}}=\mathbf{5508\sqrt{T_e}\ kms^{-1}} \\ \mathbf{k \quad = Boltzmann\ constant = 1.38\times10^{-23}\,JK^{-1}} \\ \mathbf{m_3 \quad = electron\ mass = 9.11\times10^{-31}\,kg} \\ \mathbf{T_e\ in\ 10^6\ Kelvins} \end{array}} \qquad \textbf{(A.59)}$$

However, in the rest frame of the observer, the scattered radiation off a moving electron will be altered in wavelength from the monochromatic radiation incident on that electron. Consider a coronal electron with velocity $\underset{\rightarrow}{\mathbf{u}}$ subjected to radial solar wind velocity $\underset{\rightarrow}{\mathbf{w}}$ in a coordinate system where the x-axis bisects the supplement of the scattering angle, as shown in figure (**A.8**).

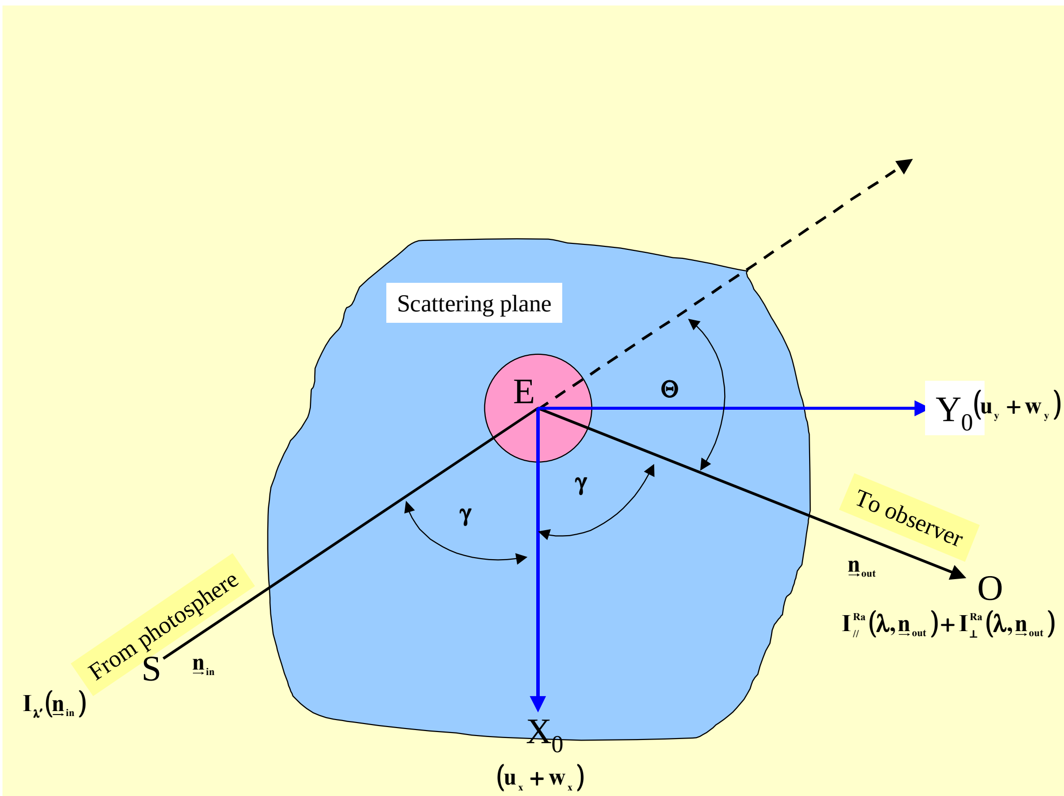

Figure (A.8). Construction to obtain an expression for the scattered intensity in the rest frame of the observer. Consider a coronal electron with velocity $\underset{\rightarrow}{\mathbf{u}}$ subjected to radial solar wind velocity $\underset{\rightarrow}{\mathbf{w}}$ in a coordinate system where the x-axis bisects the supplement of the scattering angle.

From figure (**A.8**) the direction cosines of the unit vectors $\underset{\rightarrow}{\mathbf{n}}_{\mathbf{in}}$ and $\underset{\rightarrow}{\mathbf{n}}_{\mathbf{out}}$ are then given by equation (**A.60**) and equation (**A.61**).

$$\underset{\rightarrow}{\mathbf{n}}_{\mathbf{in}} = \left(-\cos\gamma, \sin\gamma, 0\right) \tag{A.60}$$

$$\underset{\rightarrow}{\mathbf{n}}_{\mathbf{out}} = \left(\cos\gamma, \sin\gamma, 0\right) \tag{A.61}$$

And the net velocity vector $\underset{\rightarrow}{\mathbf{V}}$ is given by equation (**A.62**).

$$\underset{\rightarrow}{\mathbf{V}} = \left(\mathbf{u}_{x} + \mathbf{w}_{x}, \mathbf{u}_{y} + \mathbf{w}_{y}, \mathbf{u}_{z} + \mathbf{w}_{z}\right) \tag{A.62}$$

From Doppler effect, in the rest frame of the observer, the observed radiation has wavelength given by equation (**A.63**).

$$\lambda_{\mathbf{observed}} = \lambda_{\mathbf{scattered}} \left(\frac{1 - \frac{\underset{\rightarrow}{\mathbf{n}}_{\mathbf{out}} \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}}{1 + \frac{\underset{\rightarrow}{\mathbf{n}}_{\mathbf{out}} \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}} \right)^{\frac{1}{2}} \tag{A.63}$$

The relationship between $\lambda_{\mathbf{observed}}$ and $\lambda_{\mathbf{photosphere}}$ is given by equation (**A.64**).

$$\lambda_{\mathbf{scattered}} = \lambda_{\mathbf{photosphere}} \left(\frac{1 + \frac{\underset{\rightarrow}{\mathbf{n}}_{\mathbf{in}} \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}}{1 - \frac{\underset{\rightarrow}{\mathbf{n}}_{\mathbf{in}} \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}} \right)^{\frac{1}{2}} \tag{A.64}$$

From equation (**A.63**) and equation (**A.64**) eliminating $\lambda_{\text{scattered}}$ gives equation (**A.65**).

$$\lambda_{\text{observed}} = \lambda_{\text{photosphere}} \left(\frac{1 - \frac{\underset{\rightarrow}{\mathbf{n}}_{\text{out}} \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}}{1 + \frac{\underset{\rightarrow}{\mathbf{n}}_{\text{out}} \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}} \right)^{\frac{1}{2}} \left(\frac{1 + \frac{\underset{\rightarrow}{\mathbf{n}}_{\text{in}} \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}}{1 - \frac{\underset{\rightarrow}{\mathbf{n}}_{\text{in}} \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}} \right)^{\frac{1}{2}} \tag{A.65}$$

Since the speed of light $\mathbf{c} \gg \underset{\rightarrow}{\mathbf{V}}$ equation (**A.65**) can be reduced to equation (**A.66**).

$$\lambda_{\text{observed}} \cong \lambda_{\text{photosphere}} \left(\frac{1 + \frac{\left(\underset{\rightarrow}{\mathbf{n}}_{\text{in}} - \underset{\rightarrow}{\mathbf{n}}_{\text{out}}\right) \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}}{1 - \frac{\left(\underset{\rightarrow}{\mathbf{n}}_{\text{in}} - \underset{\rightarrow}{\mathbf{n}}_{\text{out}}\right) \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}} \right)^{\frac{1}{2}} \tag{A.66}$$

By Taylor expansion of equation (**A.66**) it could be reduced to equation (**A.67**).

$$\boxed{\begin{aligned} \lambda_{\text{observed}} &\cong \lambda_{\text{photosphere}} \left(\frac{1 + \frac{\left(\underset{\rightarrow}{\mathbf{n}}_{\text{in}} - \underset{\rightarrow}{\mathbf{n}}_{\text{out}}\right) \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}}{1 - \frac{\left(\underset{\rightarrow}{\mathbf{n}}_{\text{in}} - \underset{\rightarrow}{\mathbf{n}}_{\text{out}}\right) \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}}} \right)^{\frac{1}{2}} \\ &\cong \lambda_{\text{photosphere}} \left(1 + \frac{\left(\underset{\rightarrow}{\mathbf{n}}_{\text{in}} - \underset{\rightarrow}{\mathbf{n}}_{\text{out}}\right) \cdot \underset{\rightarrow}{\mathbf{V}}}{\mathbf{c}} \right) \end{aligned}} \tag{A.67}$$

From equation (**A.60**), (**A.61**) and (**A.62**) equation (**A.68**) could be obtained.

$$\left(\underset{\rightarrow}{\mathbf{n}}_{\mathbf{in}} - \underset{\rightarrow}{\mathbf{n}}_{\mathbf{out}}\right).\frac{\underset{\rightarrow}{\mathbf{v}}}{\mathbf{c}} = \left[-2\cos\gamma\left(\frac{\mathbf{u}_{\mathbf{x}} + \mathbf{w}_{\mathbf{x}}}{\mathbf{c}}\right)\right] \qquad \textbf{(A.68)}$$

From equation (**A.67**) and equation (**A.68**) and using the notations used in figure (**A.8**) equation (**A.69**) could be obtained, which relates between $\lambda_{\mathbf{obseved}}$ and $\lambda_{\mathbf{photosphere}}$.

$$\boxed{\begin{array}{l} \lambda = \lambda'\left(1 - 2\cos\gamma\left(\frac{\mathbf{u}_{\mathbf{x}} + \mathbf{w}_{\mathbf{x}}}{\mathbf{c}}\right)\right) \ \ \textbf{where} \\ \lambda = \lambda_{\mathbf{observed}} \\ \lambda' = \lambda_{\mathbf{photosphere}} \end{array}} \qquad \textbf{(A.69)}$$

Equation (**A.79**) is obtained by rearranging equation (**A.69**).

$$\boxed{\left(\frac{2\cos\gamma\lambda'}{\mathbf{c}}\right)\mathbf{u}_{\mathbf{x}} - \left[\lambda'\left(1 - \frac{2\cos\gamma}{\mathbf{c}}\mathbf{w}_{\mathbf{x}}\right) - \lambda\right] = 0} \qquad \textbf{(A.70)}$$

Equation (**A.70**) satisfies the condition that scattering is coherent in the rest frame of the electron.

Now, from figure (**A.8**) and equation (**A.53**), (**A.54**), (**A.59**), (**A.70**) the intensity scattered from an electron at point P is given by equation (**A.71**).

$$\begin{aligned}&I_{//}^{Ra}(\lambda, \underset{\to}{n}_{out}) + I_{\perp}^{Ra}(\lambda, \underset{\to}{n}_{out}) = I(\lambda', \underset{\to}{n}_{out}) \times \left(Q_{//}^{Ra}(\alpha,\Theta) + Q_{\perp}^{Ra}(\alpha,\Theta)\right) \times ND \\ &\text{where} \\ &ND = \frac{N_e(P)}{\left(\sqrt{\pi}q\right)^3} \int_{-\infty}^{+\infty} e^{\frac{-u^2}{q^2}} \delta\left[\left(\frac{2\cos\gamma\lambda'}{c}\right)u_x - \left(\lambda'\left(1 - \frac{2\cos\gamma}{c}w_x\right) - \lambda\right)\right] du_x du_y du_z\end{aligned} \tag{A.71}$$

The expression for **ND** in equation (**A.71**) reduces to equation (**A.72**).

$$\begin{aligned}ND &= \frac{N_e(P)}{\left(\sqrt{\pi}q\right)^3} \int_{-\infty}^{+\infty} e^{-\frac{u_y^2}{q^2}} du_y \int_{-\infty}^{+\infty} e^{-\frac{u_z^2}{q^2}} du_z \times \\ &\quad \int_{-\infty}^{+\infty} e^{-\frac{\left(\frac{2b\lambda' u_x}{c}\right)^2}{\left(\frac{2bq\lambda'}{c}\right)^2}} \delta\left[\left(\frac{2b\lambda' u_x}{c}\right) - \left(\lambda'\left(1 - \frac{2bw_x}{c}\right) - \lambda\right)\right] \frac{d\left(\frac{2b\lambda' u_x}{c}\right)}{\left(\frac{2b\lambda'}{c}\right)} \\ &= \frac{N_e(P)}{\left(\sqrt{\pi}q\right)^3} \times \left(\sqrt{\pi}q\right) \times \left(\sqrt{\pi}q\right) \times \exp\left(-\frac{\left(\lambda'\left(1 - \frac{2bw_x}{c}\right) - \lambda\right)^2}{\left(\frac{2bq\lambda'}{c}\right)^2}\right) \frac{1}{\left(\frac{2b\lambda'}{c}\right)} \\ &= \frac{N_e(P)}{\left(2\sqrt{\pi}\Delta b\right)} \exp\left[-\left(\frac{\lambda - \lambda'\left(1 - \frac{2bw_x}{c}\right)}{(2\Delta b)}\right)^2\right] \\ &\text{where } \Delta = \frac{q\lambda'}{c} \text{ and } b = \cos\gamma = \cos\left(\frac{\pi - \Theta}{2}\right)\end{aligned} \tag{A.72}$$

From equation (**A.71**) and (**A.72**) the scattered intensity from point P is given by equation (**A.73**).

$$\mathbf{I_O^{Ra}\left(\lambda,\underset{\rightarrow}{n}_{out}\right)=I\left(\lambda',\underset{\rightarrow}{n}_{in}\right)\times Q_O^{Ra}\left(\alpha,\Theta\right)\times\frac{N_e\left(P\right)}{2\sqrt{\pi}\Delta b}\exp\left[-\left(\frac{\lambda-\lambda'\left(1-\frac{2bw_x}{c}\right)}{2\Delta b}\right)^2\right]} \qquad \textbf{(A.73)}$$

where $\mathbf{O\equiv\left(//,\perp\right)}$

To obtain an expression for the total observed scattered intensity, from figure (**A.6**), the equation (**A.73**) needs to be integrated over the following parameters.

1. All wavelengths from each point on the photosphere

$$\int_0^{\infty}\mathbf{d\lambda'} \qquad \textbf{(A.74)}$$

2. From all points on the photosphere

$$\int_0^{2\pi}\mathbf{d\varphi}\int_0^{\omega^*}\mathbf{\sin\omega\, d\omega\equiv}\int_0^{2\pi}\mathbf{d\varphi}\int_{\cos\omega^*}^{1}\mathbf{d\cos\omega} \qquad \textbf{(A.75)}$$

3. From all points along the line of sight

$$\int_{-\infty}^{+\infty}\mathbf{dx} \qquad \textbf{(A.76)}$$

From figure (**A.6**) and figure (**A.8**) the radial solar wind velocity has components given by equation (**A.77**).

$$\underset{\rightarrow}{\mathbf{w}} = \left(-\cos\omega\cos\gamma, \cos\omega\sin\gamma, \sin\omega\right) \equiv \left(\mathbf{w}_x, \mathbf{w}_y, \mathbf{w}_z\right) \tag{A.77}$$

Using equation (**A.74**), (**A.76**), (**A.76**) and (**A.77**) in (**A.73**) gives the expression for the total observed intensity for a given observed wavelength λ at a given line of sight distance ρ from the center of the Sun (see figure (**A.6**)), as given by equation (**A.78**).

$$\boxed{\begin{aligned} \mathbf{I}_{\mathbf{O}}^{\mathbf{Ra}}(\lambda,\rho) = & \int_{-\infty}^{+\infty}\int_{0}^{2\pi}\int_{\cos\omega^*}^{1}\int_{0}^{\infty} \mathbf{d}\lambda'\,\mathbf{d}\cos\omega\,\mathbf{d}\varphi\,\mathbf{dx}\times \\ & \mathbf{N}_e(\mathbf{r})\times\mathbf{Q}_{\mathbf{O}}^{\mathbf{Ra}}(\alpha,\Theta)\times \\ & \frac{1}{2\sqrt{\pi}\Delta\mathbf{b}}\mathbf{I}\left(\lambda',\underset{\rightarrow}{\mathbf{n}}_{\mathbf{in}}\right)\exp\left[-\left(\frac{\lambda-\lambda'\left(1+\dfrac{2\mathbf{b}^2\cos\omega\mathbf{w}(\mathbf{r})_{\mathbf{radial}}}{\mathbf{c}}\right)}{2\Delta\mathbf{b}}\right)^2\right] \\ & \text{where } \mathbf{O}\equiv(//,\perp) \\ & //\ \text{parallel to the radial plane} \\ & \perp\ \text{perpendicular to the radial plane} \end{aligned}} \tag{A.78}$$

A.5 Expressions for the dependent variables in equation (A.78) in terms of the independent variables

Figure (**A.9**) is a highlight from figure (**A.6**) of the triangles formed by SEW (scattering plane), CEF (radial plane) and CSE.

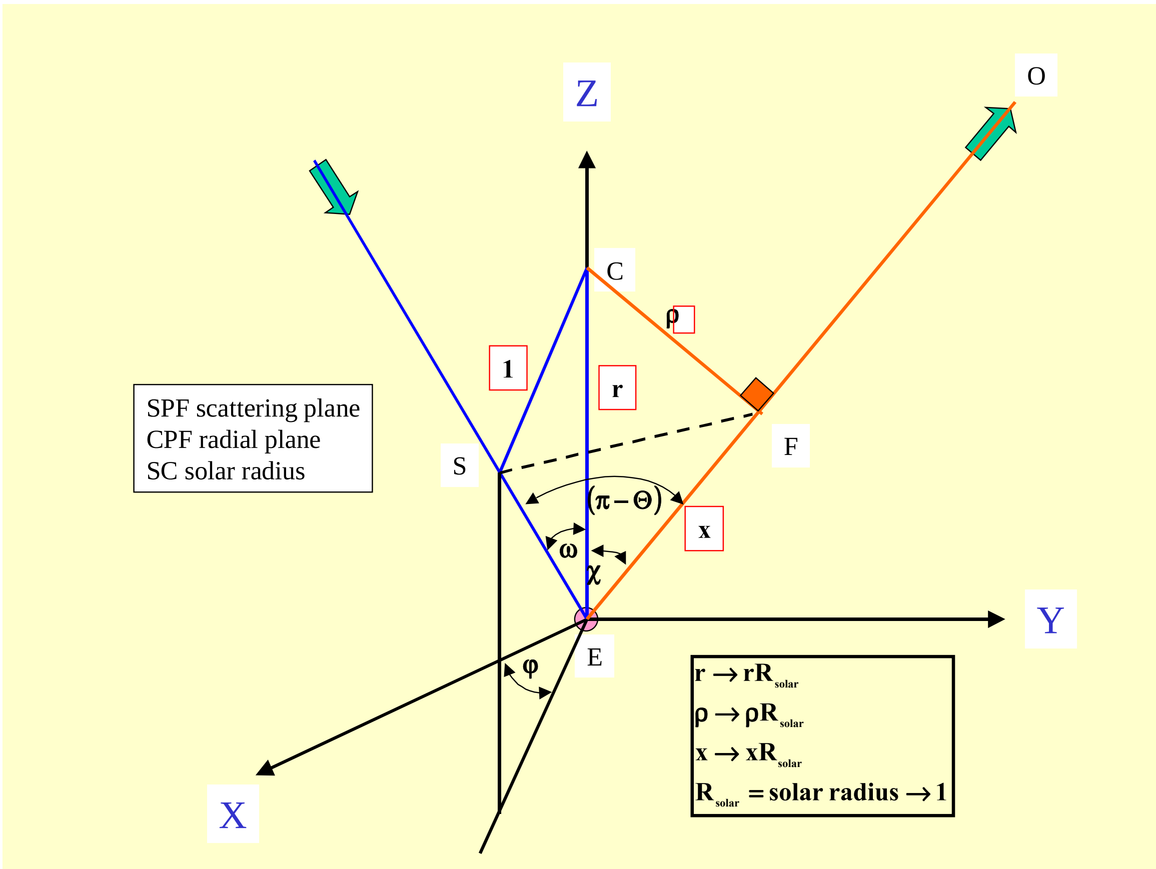

Figure (A.9). Highlighted map of the radial and the scattering planes of figure (A.6).

From figure (**A.9**) equation (**A.79**) and equation (**A.80**) can be obtained.

$$\underset{\rightarrow}{\mathbf{ES}} = \left|\underset{\rightarrow}{\mathbf{ES}}\right|(\sin\omega\cos\varphi, \sin\omega\sin\varphi, \cos\omega) \qquad \textbf{(A.79)}$$

$$\underset{\rightarrow}{\mathbf{EF}} = \left|\underset{\rightarrow}{\mathbf{EF}}\right|(0, \sin\chi, \cos\chi) \qquad \textbf{(A.80)}$$

And the dot product of equation (**A.79**) and equation (**A.80**) gives equation (**A.81**).

$$\underrightarrow{\mathbf{ES}}.\underrightarrow{\mathbf{EF}} = \left|\underrightarrow{\mathbf{ES}}\right|\left|\underrightarrow{\mathbf{EF}}\right|(\sin\omega\sin\varphi\sin\chi + \cos\omega\cos\chi) \\ = \left|\underrightarrow{\mathbf{ES}}\right|\left|\underrightarrow{\mathbf{EF}}\right|\cos(\pi - \Theta) \tag{A.81}$$

From equation (**A.81**) the expression for Θ is given by equation (**A.82**).

$$\Theta = \pi - \cos^{-1}(\sin\omega\sin\varphi\sin\chi + \cos\omega\cos\chi) \tag{A.82}$$

Also from figure (**A.9**) the expression for angle χ is given by equation (**A.83**).

$$\cos\chi = \frac{x}{r} \tag{A.83}$$

From figure (**A.6**) and measuring distances in solar radius the expression for angle ω^* is given by equation (**A.84**).

$$\sin\omega^* = \frac{HC}{EC} = \frac{1}{r} \tag{A.84}$$

From figure (**A.6**) consider the spherical triangle formed by EDJI, as shown in figure (**A.10**).

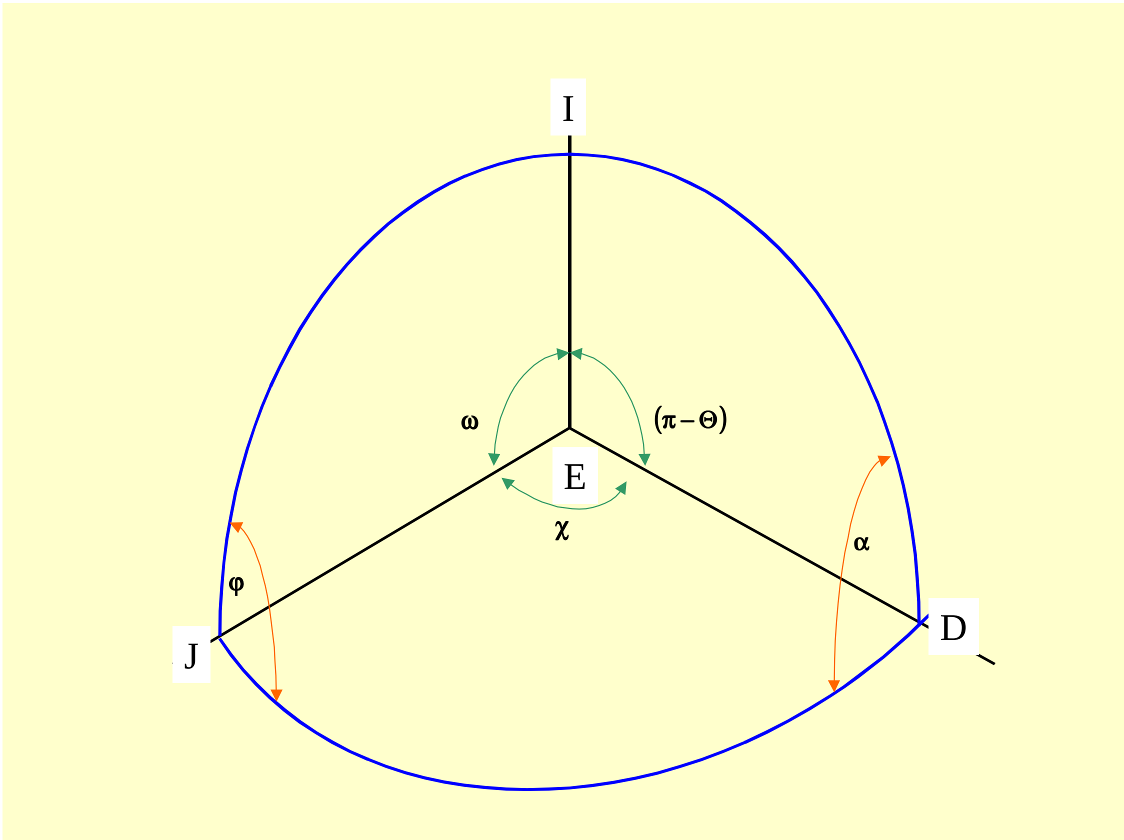

Figure (A.10). Highlight of the spherical triangle formed by EDJI in figure (A.6).

From figure (**A.10**) the relationship between the angles is given by equation (**A.85**).

$$\frac{\sin\alpha}{\sin\omega} = \frac{\sin\varphi}{\sin(\pi - \Theta)} \qquad \textbf{(A.85)}$$

From equation (**A.85**) the expression for angle α is given by equation (**A.86**).

$$\alpha = \sin^{-1}\left(\frac{\sin\omega\sin\varphi}{\sin(\pi-\Theta)}\right) \quad \textbf{(A.86)}$$

Figure (**A.11**) shows the triangle ECS from figure (**A.6**).

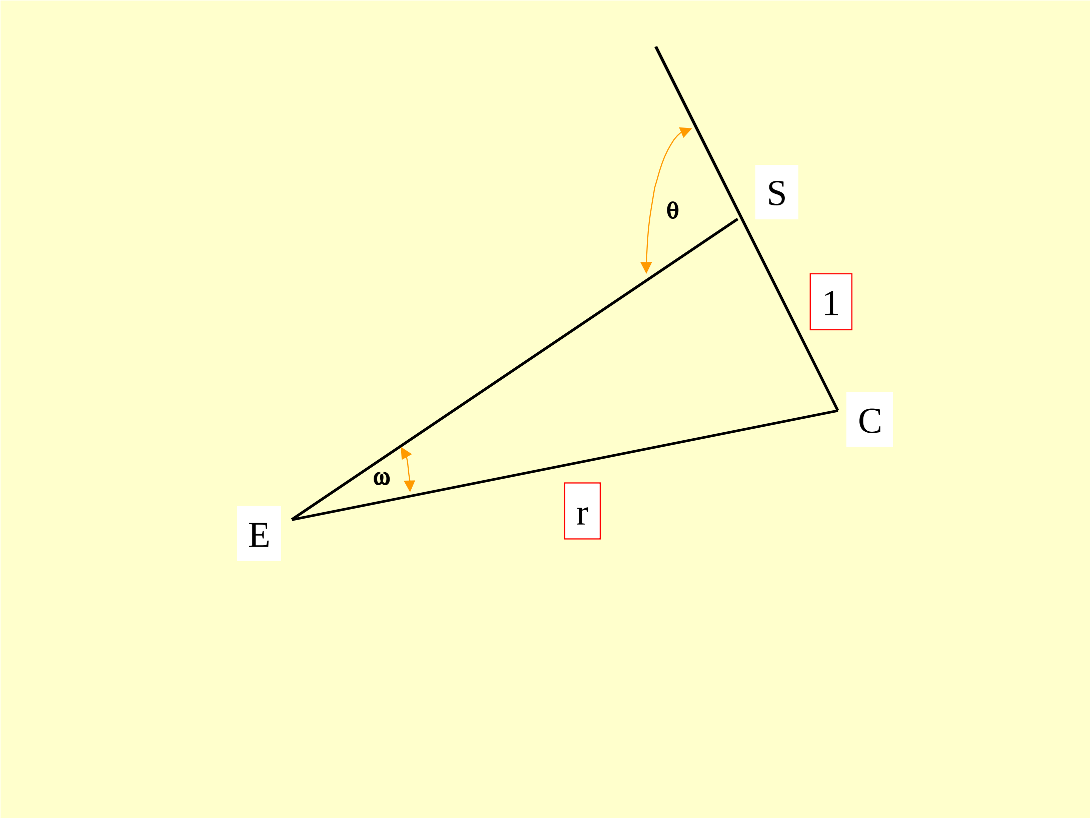

Figure (A.11). Highlight of the triangle ESC of figure (A.6).

From figure (**A.11**) the relationship between the angles θ and ω is given by equation (**A.87**).

$$\frac{1}{\sin\omega} = \frac{r}{\sin(\pi-\theta)} \quad \textbf{(A.87)}$$
$$\therefore \sin\theta = r\sin\omega$$

A.6 Expression for the incident solar intensity on the coronal electrons

In order to evaluate equation (**A.78**) the incident solar intensity $\mathbf{I}(\lambda', \underline{\mathbf{n}}_{\mathbf{in}})$ need to be known. The following expressions are from ***Astrophysical Quantities*** by ***Allen***. Let $\mathbf{I}(\lambda', \theta)$ be the intensity of the solar continuum at an angle θ from the center of the disk and $\mathbf{I}(\lambda', \mathbf{0})$ the continuum intensity at the center of the disk. The results may be fitted by the expression given by equation (**A.88**).

$$\frac{\mathbf{I}(\lambda',\theta)}{\mathbf{I}(\lambda',0)} = 1 - u_2 - v_2 + u_2 \cos\theta + v_2 \cos^2\theta \tag{A.88}$$

where u_2 and v_2 are limb darkening constants

Or less accurately equation (**A.88**) can be written as equation (**A.89**).

$$\frac{\mathbf{I}(\lambda',\theta)}{\mathbf{I}(\lambda',0)} = 1 - u_1 + u_1 \cos\theta \tag{A.89}$$

where $\cos\theta$ is the heliocentric angle shown in figure (**A.6**).

And for determining the value of $\mathbf{u_1}$ it is preferable to make a fit at $\mathbf{\cos\theta = 0.5}$, which gives the expression given by equation (**A.90**).

$$\boxed{\begin{aligned}
&\cos^2\theta = \frac{1}{2}(3\cos\theta - 1)\,\text{at}\,\cos\theta = 0.5\ \text{and}\\
&\frac{I(\lambda',\theta)}{I(\lambda',0)} = 1 - u_2 - v_2 + u_2\cos\theta + v_2\frac{1}{2}(3\cos\theta - 1)\\
&\qquad\qquad = 1 - \left(u_2 + \frac{3}{2}v_2\right) + \left(u_2 + \frac{3}{2}v_2\right)\cos\theta\\
&\qquad\qquad \equiv 1 - u_1 + u_1\cos\theta \quad \text{where}\\
&u_1 = \left(u_2 + \frac{3}{2}v_2\right)
\end{aligned}} \tag{A.90}$$

The ratio of mean to central intensity is given by equation (**A.91**).

$$\boxed{\begin{aligned}
\frac{F(\lambda')}{I(\lambda',0)} &= 1 - \frac{1}{3}u_2 - \frac{1}{2}v_2\\
&= 1 - \frac{1}{3}\left(u_2 + \frac{3}{2}v_2\right)\\
&= 1 - \frac{1}{3}u_1
\end{aligned}} \tag{A.91}$$

From equation (**A.90**) and (**A.91**) the ratio of $\mathbf{I(\lambda',\theta)/F(\lambda')}$ is given by equation (**A.92**).

$$\frac{I(\lambda',\theta)}{F(\lambda')} = \frac{1 - u_1 + u_1\cos\theta}{1 - \frac{1}{3}u_1} \tag{A.92}$$

The emittance of solar surface per unit wavelength range is given by equation (**A.93**).

$$\mathbf{E}(\lambda') = \pi \mathbf{F}(\lambda') \tag{A.93}$$

And the solar flux outside the Earth atmosphere per unit area and wavelength range is given by equation (**A.94**).

$$\boxed{\begin{aligned} \mathbf{f}(\lambda') &= \mathbf{E}(\lambda')\left(\frac{\mathbf{R}_{\mathbf{solar}}}{\mathbf{AU}}\right)^2 \\ &= \pi\left(\frac{\mathbf{R}_{\mathbf{solar}}}{\mathbf{AU}}\right)^2 \mathbf{F}(\lambda') \end{aligned}} \tag{A.94}$$

From equation (**A.90**) and equation (**A.92**) the incident intensity on the coronal electrons is given by equation (**A.95**).

$$\boxed{\mathbf{I}(\lambda', \theta) = \left(\frac{1}{\pi}\right)\frac{1 - \mathbf{u}_1 + \mathbf{u}_1 \cos\theta}{1 - \frac{1}{3}\mathbf{u}_1}\left(\frac{\mathbf{AU}}{\mathbf{R}_{\mathbf{solar}}}\right)^2 \mathbf{f}(\lambda')} \tag{A.95}$$

In equation (**A.94**) and equation (**A.95**) AU is the Sun-Earth distance. The wavelength dependent limb-darkening coefficient $\mathbf{u}_1$ can be obtained from ***Astrophysical Quantities*** by ***Allen***

A.7 Final expression for the observed intensity

From figure (**A.6**) and equation (**A.78**) all the following expressions are given in terms of solar radii, as shown in equation (**A.96**).

$$\boxed{\begin{array}{l} \mathbf{r \rightarrow rR_{solar}} \\ \boldsymbol{\rho \rightarrow \rho R_{solar}} \\ \mathbf{x \rightarrow xR_{solar}} \end{array}} \qquad \mathbf{(A.96)}$$

And the observed intensity is given by equation (**A.97**).

$$\boxed{\begin{array}{l} I_O^{Ra}\left(\lambda, \rho R_{solar}\right) = \int\limits_{-\infty}^{+\infty}\int\limits_{0}^{2\pi}\int\limits_{\cos\omega^*}^{1}\int\limits_{0}^{\infty} d\lambda'\, d\varphi\, d\cos\omega\;\; d(xR_{solar})\times \\ \qquad N_e\left(rR_{solar}\right)\times Q_O^{Ra}\left(\alpha,\Theta\right)\times \\ \qquad \dfrac{1}{2\sqrt{\pi}\Delta b} I\left(\lambda',\omega,x\right)\exp\left[-\left(\dfrac{\lambda-\lambda'\left(1+\dfrac{2b^2\cos\omega w\left(rR_{solar}\right)_{radial}}{c}\right)}{2\Delta b}\right)^2\right] \\ \text{where } O \equiv \left(//,\perp\right) \\ //\ \text{parallel to the radial plane} \\ \perp\ \text{perpendicular to the radial plane} \end{array}} \qquad \mathbf{(A.97)}$$

The expressions for physical parameters in equation (**A.97**) in terms of independent variables are given in equation (**A.98**). Equation (**A.99**) gives the parameters in equation (**A.97**) for which suitable models or actual measurements need to be used.

$$
\begin{aligned}
&Q_{//}^{Ra} = \frac{3}{16\pi}\sigma_T\left(\sin^2\alpha + \cos^2\alpha\cos^2\Theta\right)\\
&Q_{\perp}^{Ra} = \frac{3}{16\pi}\sigma_T\left(\cos^2\alpha + \sin^2\alpha\cos^2\Theta\right)\\
&b = \cos\gamma = \cos\left(\frac{\pi-\Theta}{2}\right)\\
&\Delta = \frac{q\lambda'}{c}\\
&q = \sqrt{\frac{2kT}{m}}\\
&I(\lambda',\omega,x) = \frac{1}{\pi}\left(\frac{AU}{R_{solar}}\right)^2\left(\frac{1-u_1+u_1\cos\theta}{1-\frac{1}{3}u_1}\right)f\\
&\Theta = \pi - \cos^{-1}\left(\sin\omega\sin\varphi\sin\chi + \cos\omega\cos\chi\right)\\
&\alpha = \sin^{-1}\left(\frac{\sin\omega\sin\varphi}{\sin(\pi-\Theta)}\right)\\
&\chi = \cos^{-1}\left(\frac{x}{r}\right)\\
&\omega^* = \sin^{-1}\left(\frac{1}{r}\right)\\
&\theta = \sin^{-1}\left(r\sin\omega\right)\\
&r^2 = x^2 + \rho^2
\end{aligned}
\tag{A.98}
$$

$$
\begin{aligned}
&u_1(\lambda') = \text{limb darkening coefficient}\\
&f(\lambda') = \text{extraterrestrial solar irradiance}\\
&N_e(rR_{solar}) = \text{electron density model}\\
&T(rR_{solar}) = \text{coronal temperature model}\\
&W(rR_{solar}) = \text{solar wind model}
\end{aligned}
\tag{A.99}
$$